

Impulsive Model for *Wolbachia*-Based Control Under Thermal Stress

Modelo Impulsivo para o Controle com *Wolbachia* sob Estresse Térmico

Modelo impulsivo para el control con *Wolbachia* bajo estrés térmico



Jéssica Correia Santos Alves

Universidade de São Paulo

Joint work with:

Cláudia Pio Ferreira (UNESP)

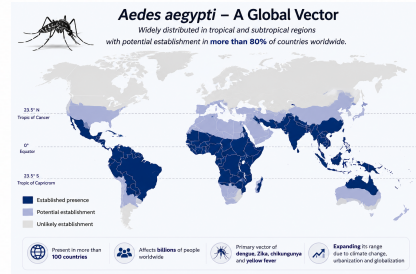
Christian Emilio Schaerer Serra (UNA)

Based on: "Impulsive Release Strategies for *Wolbachia*-Infected Mosquitoes Under Temperature-Induced Infection Loss", Bulletin of Mathematical Biology, 2026.

Motivation: *Aedes aegypti* and *Wolbachia*

The public health challenge

- *Aedes aegypti* is a major vector of arboviruses, including:
 - dengue;
 - Zika;
 - chikungunya.
- Its geographic expansion increases the potential risk of arbovirus transmission.



Global distribution of *Aedes aegypti*.

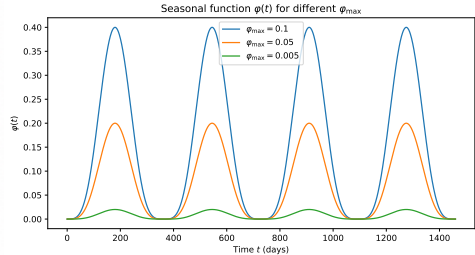
Wolbachia-based control

Wolbachia-infected mosquitoes can be released to reduce vector competence and promote population replacement.

The Thermal-Stress Problem

Temperature and *Wolbachia*

- Environmental temperature can affect *Wolbachia* infection dynamics.
- High temperatures may reduce *Wolbachia* density within the mosquito.
- Reduced infection can compromise the establishment and persistence of *Wolbachia*.



Seasonal variation in infection loss.

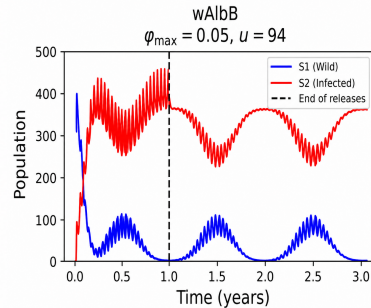
The challenge

How can *Wolbachia* infection be maintained under thermal stress?

Impulsive Intervention Strategy

Why impulsive interventions?

- Releases of infected mosquitoes occur at **discrete times**.
- Thermal stress produces a **time-dependent loss of infection**.
- The model represents both effects through **impulsive events**.
- Infection loss transfers mosquitoes to the wild population, while periodic releases increase the infected population.



Population dynamics under periodic releases.

Impulsive Mathematical Model

Continuous dynamics

$$t \neq (n+1)\tau, \quad t \neq (n+1)\tau$$

$$\begin{cases} \frac{dS_1}{dt} = S_1 \psi_1 \left(1 - \frac{S_1 + S_2}{K} \right) \left(\frac{S_1 + (1-\gamma)S_2}{S_1 + S_2} \right) - \delta_1 S_1, \\ \frac{dS_2}{dt} = S_2 \psi_2 \left(1 - \frac{S_1 + S_2}{K} \right) - \delta_2 S_2 \end{cases}$$

Impulsive effects

Infection loss

$$t = (n+1)\tau$$

$$\begin{cases} S_1(t^+) = S_1(t) + \varphi(t)S_2(t), \\ S_2(t^+) = (1 - \varphi(t))S_2(t) \end{cases}$$

Release

$$t = (n+1)\tau$$

$$\begin{cases} S_1(t^+) = S_1(t), \\ S_2(t^+) = S_2(t) + u_n \end{cases}$$

Populations

$S_1(t)$: adult wild
Aedes aegypti

$S_2(t)$: adult
Wolbachia-infected
Aedes aegypti

- γ : CI;
- $\varphi(t)$: infection loss;
- u_n : release size;
- $l \in [0, 1]$: relative timing of infection loss.

Based on: "Impulsive Release Strategies for Wolbachia-Infected Mosquitoes Under Temperature-Induced Infection Loss", Bulletin of Mathematical Biology, 2026.

Temperature-Driven Infection Loss

Seasonal infection loss

$$\varphi(t) = \varphi_{\max} \left[1 + \cos \left(\frac{2\pi(t - t_{\text{peak}})}{365} \right) \right]^2$$

- Periodic seasonal forcing;
- $\varphi_{\max}$ controls the intensity of infection loss;
- $\varphi(t) \in [0, 4\varphi_{\max}]$.

Thermal effect

Higher $\varphi(t)$ means a larger fraction of *Wolbachia*-infected mosquitoes loses the infection.

Temperature-induced infection loss acts as a **periodic environmental disturbance** within the impulsive model.

Periodic Solutions

Theorem

Let $(S_1(0), S_2(0))$ be non-negative initial conditions. Assume that, in the absence of redistributive impulses (i.e., $\varphi \equiv 0$), the system admits a unique, positive, τ -periodic, wild-mosquito-free solution

$$(0, \bar{S}_2(t)).$$

Then, for φ sufficiently small, the impulsive system admits a unique, positive, τ -periodic solution

$$(S_1^\varphi(t), \bar{S}_2^\varphi(t)),$$

where $S_1^\varphi(t)$ remains uniformly small and $\bar{S}_2^\varphi(t)$ stays uniformly close to $\bar{S}_2(t)$ for all $t \geq 0$.

Stability of the Periodic Solution

Theorem

Suppose the hypotheses of the previous theorem hold and, moreover, that the τ -periodic wild-mosquito-free solution $(0, \bar{S}_2(t))$ satisfies

$$\bar{S}_2(t) > K \left(1 - \frac{\delta_1}{\psi_1} \right), \quad \text{for all } t \in \mathbb{R}.$$

Then:

- (i) The periodic orbit $(0, \bar{S}_2(t))$ of the unperturbed system ($\varphi \equiv 0$) is globally asymptotically stable in $\mathbb{R}_+^2$.
- (ii) For sufficiently small $\varphi_{\max}$, the perturbed periodic solution is asymptotically stable, with

$$s_1^\varphi(t) \rightarrow 0, \quad \bar{s}_2^\varphi(t) \rightarrow \bar{S}_2(t),$$

uniformly on $[0, \tau]$ as $\varphi_{\max} \rightarrow 0$.

A Sufficient Threshold for Release Size

Stability Requirement

For the infected population to maintain the desired regime, it is sufficient to have

$$\bar{S}_2(t) > \hat{K}, \quad \hat{K} = \kappa \left(1 - \frac{\delta_1}{\psi_1} \right).$$

Uniform Sufficient Bound

Since $\eta(\tau)$ depends on τ , a conservative condition is

$$u_n > \max_{\tau > 0} \eta(\tau).$$

The existence of this global maximum is guaranteed analytically.

Simulation Setup

Strain-specific parameters

Strain	α_ψ	α_δ
wMelPop	0.50	1.50
wMel	0.95	1.10
wAlbB	0.85	1.00

$$\psi_2 = \alpha_\psi \psi_1, \quad \delta_2 = \alpha_\delta \delta_1.$$

Thermal infection-loss scenarios

	wMelPop	wMel	wAlbB
Low	0.0015	0.001	0.0005
Moderate	0.015	0.010	0.005
High	0.150	0.100	0.050

Simulation protocol

- Releases during the first 365 days.
- Additional 730 days without releases.
- Total simulation: 3 years.
- Release intervals: $\tau = 7, 14$ days.

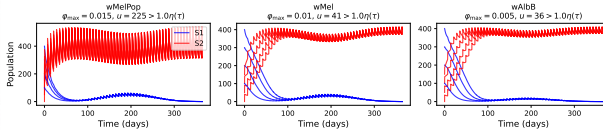
Thermal sensitivity

$$w\text{MelPop} > w\text{Mel} > w\text{AlbB}$$

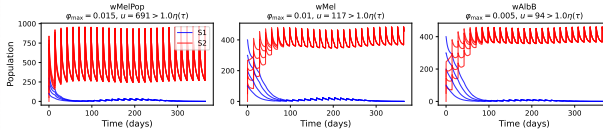
Exploratory infection-loss scenarios; not field-calibrated predictions.

Effect of Release Interval Under Moderate Thermal Stress

Release interval: $\tau = 7$ days



Release interval: $\tau = 14$ days

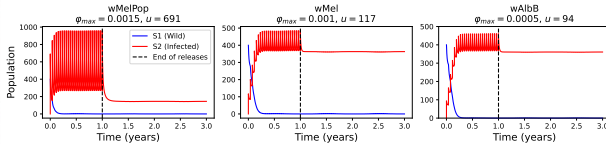


Main findings

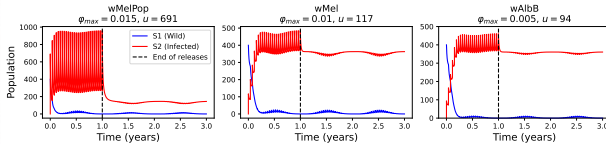
- All three strains establish under both release intervals.
- Increasing τ affects the transient population dynamics.
- wMelPop exhibits larger oscillations and slower wild population suppression.
- wMel and wAlbB show comparatively more stable dynamics.

Persistence Under Low and Moderate Thermal Stress

Low infection loss



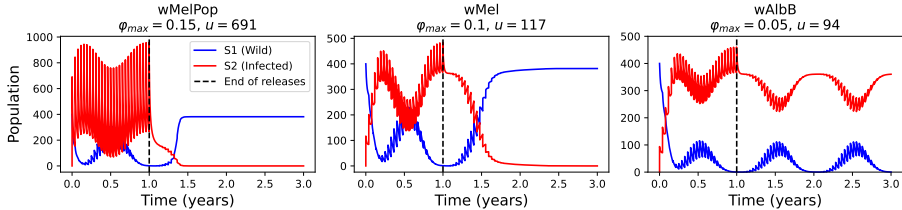
Moderate infection loss



Main findings

- All three strains maintain a positive infected population under low and moderate thermal stress.
- Wild mosquitoes are suppressed throughout the three-year simulation.
- Moderate thermal stress leads to stronger seasonal oscillations.

High Thermal Stress: Strain-Dependent Persistence



wMelPop and wMel

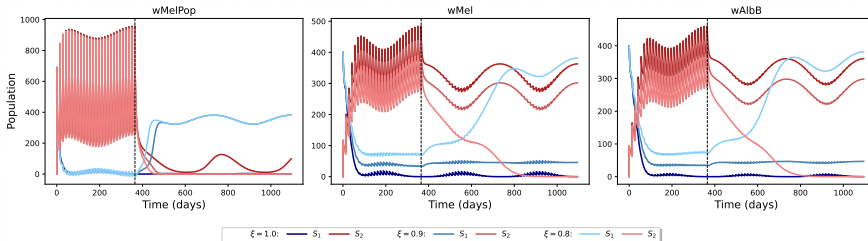
In the simulated high-loss scenario, infection is lost after releases cease, and the wild population recovers.

wAlbB

In the same scenario, the infected population remains established despite stronger seasonal fluctuations.

Robustness Under Combined Effects

Temperature-dependent demography and imperfect maternal transmission (ξ)



Numerical extension beyond the analytical framework.

Maternal Transmission

- $\xi = 0.9$: wMel and wAlbB persist.
- $\xi = 0.8$: all strains lose persistence.

Main Conclusion

Combined thermal and transmission effects can reverse population replacement, leading to wild population recovery.

Main Findings and Limitations

Main findings

- Analytical conditions characterize periodic solutions and release thresholds.
- Fitness costs and thermal stress shape strain-specific persistence.
- wAlbB exhibits greater resilience in the simulated high-stress scenarios.
- Imperfect transmission can compromise establishment under combined effects.

Model limitations

- Adult mosquitoes only.
- Temperature affects infection loss in the analytical model, but not demographic rates.
- Perfect maternal transmission in the analytical framework.
- Extended biological mechanisms explored numerically.

Analytical and numerical approaches complement each other in assessing Wolbachia establishment.

Related work

Impulsive Release Strategies for Wolbachia-Infected Mosquitoes Under Temperature-Induced Infection Loss

Bulletin of Mathematical Biology (2026)



Scan to read the full paper

DOI: 10.1007/s11538-026-01761-w

¡Gracias!



This study was financed, in part, by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) – Finance Code 001.